Markov Chain Model for Microcredit Leading to Inclusion

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1. Introduction

Microredit is *a low amount credit bearing interests* granted to *micro-entrepreneurs* who *do not have access to regular bank services.*

This type of credit makes it possible for people who are *excluded from the banking system* as we know it to *create or develop an income-generating activity.* It can be granted to *individuals* as well as *groups.*

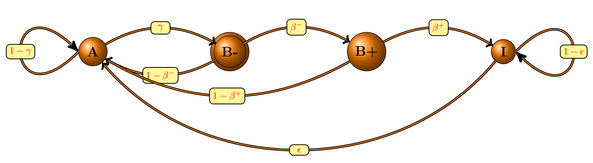
Previous mathematical models of microcredit: (Tedeschi[2006]; Khodr, M. Diener, F. Diener et Protter[2009]; Dhib, M. Diener et F. Diener[2013])

The purpose of the research: find a new mathematics models for analyze the other important characteristic of microcredit, namely the mechanism that allows a beneficiary with *some successive microcredit* can *access a credit from regular bank.*

2. Markov Chain Model

In the model, we introduce a **Markov chain** $(X_t)_{t\geq 0}$, which consist of four states:

- A (applicant)
- B^- (beneficiary with a loan k^-, an interest rate r)
- B^+ (beneficiary with a loan k^+, an interest rate r, $k^+ > k^-$)
- I (inclu with a loan k^+, an interest rate r', $r' < r$)



The transition matrix:

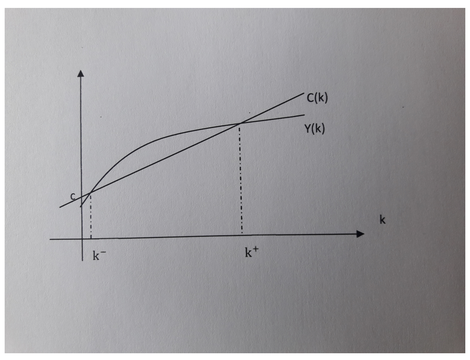
$$P = \begin{pmatrix} 1-\gamma & \gamma & 0 & 0 \\ 1-\beta^- & 0 & \beta^- & 0 \\ 1-\beta^+ & 0 & 0 & \beta^+ \\ \epsilon & 0 & 0 & 1-\epsilon \end{pmatrix}$$

Example of a trajectory:

- $AB^-B^+IIIIII...$
- $AAB^-AB^-B^+IIA...$

3. Net Revenue

Net Revenue is the production function minus the cost function
$Y(k)$ is the production function.
$C(k) = c + (1+r)k$ is the cost function.



$Y(k) = k^\alpha$
$C(k) = c + (1+r)k$
$(Y-C)(k^-) = (Y-C)(k^+) = 0$

4. The Expected Total Profit

- **The profit of one period**

$$f(X_{t-1}, X_t) = \begin{cases} (r-r')k^+ & \text{if } (I,I) \\ 0 & \text{if not} \end{cases}$$

- **The total discounted profit of a trajectory** $(X_0, X_1, ..., X_t, ...)$ for $X_0 = x$

$$W(x) = \sum_{t\geq 1} \delta^t f(X_{t-1}, X_t)$$

δ is discount factor for a time of a loan.

- **Proposition (Dhib/Diener)**

$$\mathbb{E}(W(x)) = \delta(I - \delta P)^{-1} Z(x)$$

where $Z(x) = \mathbb{E}(f(X_0, X_1)|X_0 = x)$.

5. Resultats

- **Proposition (application of the Perron-Frobenius theorem)**
Let $(X_t)_{t\geq 0}$ be the above Markov chain with the state space $S = \{A, B^-, B^+, I\}$ and transition matrix P. The Markov chain has a unique equilibrium state π^*,

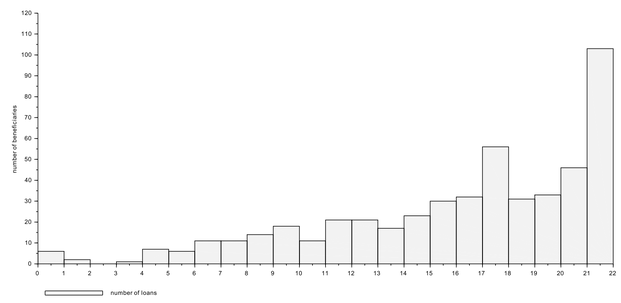
$$\pi^* = \frac{1}{\epsilon + \epsilon\gamma + \epsilon\gamma\beta^- + \gamma\beta^-\beta^+}(\epsilon, \epsilon\gamma, \epsilon\gamma\beta^-, \gamma\beta^-\beta^+).$$

The proportions of four states tends to the equilibrium distribution π^*, when t tends $+\infty$.

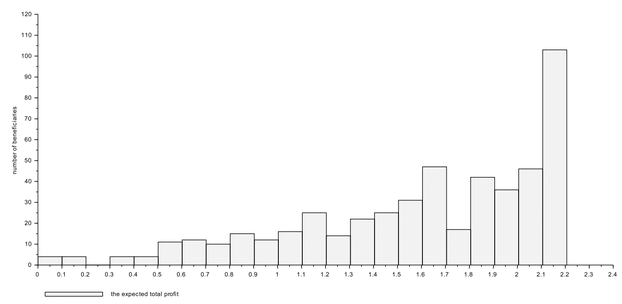
- **Parameter estimation**
If the MFI has type data $(X_t)_{t=0,...,+\infty}$ then the Ergodic theorem would allow it to calculate π^{X_t} and then γ, β^- and β^+ :
$\gamma = \frac{\pi^*_{B^-}}{\pi^*_A}$; $\beta^- = \frac{\pi^*_{B^+}}{\pi^*_{B^-}}$; $\frac{\beta^+}{\epsilon} = \frac{\pi^*_I}{\pi^*_{B^+}}$.

- **Simulation of the distribution of total profit**

  – **Number of loans (*I*, *I*)**
  Number of applicants/beneficiaries=500; Trajectory length=25; γ=0.5, β^-=0.9, β^+=0.7, ϵ=0.02.



  – **Total discounted profit**
  Number of applicants/beneficiaries=500; Trajectory length=25; γ=0.5, β^-=0.9, β^+=0.7, ϵ=0.02, r=0.2, r'=0.04.

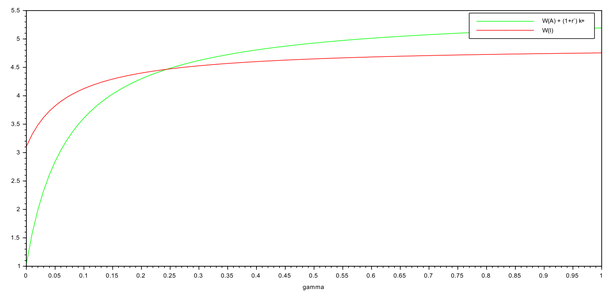


- **Absence of strategic default**
To avoid a strategic default, the MFI has a make sur the parameters have been fixed so that:

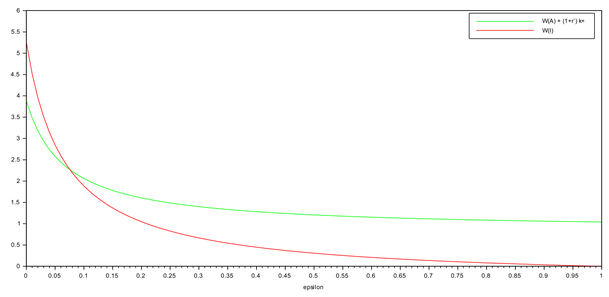
$$W_t(I) \geq W_t(A) + (1+r')k^+$$

- **Application of the proposition (Dhib/Diener)**
We apply the proposition (Dhib/Diener) to search the values γ and ϵ at the condition of absence of strategic default.

  – Case 1:
  β^-=0.9, β^+=0.7, ϵ=**0.02**, r=0.2, r'=0.04



  – Case 2:
  γ=**0.1**, β^-=0.9, β^+=0.7, r=0.2, r'=0.04



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[2] F. Diener, M. Diener, O. Khodr, P. Protter. Mathematical Models for Microlending. *Proceedings of the 16th Mathematical Conference of Bangladesh Mathematical Society, Dhaka, Bangladesh*, 2009.

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Poster Presentations

WU Vienna, Mon, September 9, 2019, 10:40 (first coffee break)
to Tue, September 10, 2019, 11:10 (morning coffee break)
LC Forum, LC - Learning Center, ground floor (LC.0.000)

Tereza Cristina **Amorelli** (Banco do Brasil)
Pricing non-traded assets using indifference pricing

Alejandro **Balbás** (University Carlos III of Madrid/Spain)
Golden strategies in derivative markets

Erwinna **Chendra** (Parahyangan Catholic University)
Pricing employee stock options with a binomial method: case study in indonesia

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Ewa **Dziwok** (University of Economics in Katowice)
Fund Transfer Pricing mechanism – different approaches to the reference yield's construction

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Alireza **Fallahi** (Amirkabir University of Technology)
Sufficient nonlinear forecasting using factor models

Pavel V. **Gapeev** (London School of Economics)
On the Fourier-Laplace transforms of first exit times for one-dimensional diffusions and their applications to models of stochastic volatility

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Laura **Garcia-Jorcano** (Universidad de Castilla-La Mancha)
Traffic light system for systemic stress: TALIS-cube

Ivana **Geček Tuđen** (University of Zagreb)
Ruin probability for discrete risk processes

Darjus **Hosszejni** (WU Vienna)
Approaches toward the Bayesian estimation of the stochastic volatility Model with leverage

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Verena **Köck** (WU Wien)
Option hedging in models with jumps

Borys **Koval** (Vienna University of Economics and Business)
Estimating a time-varying parameter model with shrinkage for the Standard&Poor's 500 index.

Djaffar **Lessy** (Université Cote d'Azur)
Markov chain model for microcredit leading to inclusion

Paul Felix **Reiter** (TU Dresden)
Feature engineering in univariate time series forecasting

Anne **Sumpf** (Technische Universität Dresden)
Credit Risk with Credibility Theory: a distribution-free estimator for probability of default, value-at-risk and expected shortfall

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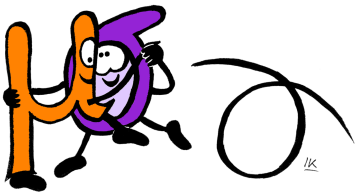

Confirmation of Presentation

Vienna, September 16, 2016

To Whom it May Concern:

We herewith confirm that

Djaffar LESSY
(Université Cote d'Azur, France)

presented the following poster

"Markov chain model for microcredit leading to inclusion"

at the

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Mon–Wed, Sept. 9-11, 2019.

With best regards,

Sandra Trenovatz
VCMF Conference & Workshop Secretary

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